References

[1]Calcote, H. P., "Electrical Properties of Flames," *Third Symposium on Combustion, Flame, and Explosion Phenomena*, Baltimore, 1949, pp. 245-253.

[2]Calcote, H. P. and Pease, R. N., "Electrical Properties of Flames," *Industrial Engineering Chemicals*, Vol. 43, 1951, p. 2726.

[3]Harker, J. H. and Porter, J. E., "The Enhancement of the Stability of Methane Diffusion Flame by the Application of Dielectrical Fields," *Journal of Institute of Fuels*, Vol. 41, 1968, p. 264.

[4]Jaggers, H. C. and von Engel, A., "The Effect of Electric Fields on the Burning Velocity of Various Flames," *Combustion and Flame*, Vol. 16, 1971, pp. 275-285.

[5]Bowser, R. J. and Weinberg, F. J., "The Effect of Direct Electric Fields on Normal Burning Velocity," *Combustion and Flame*, Vol. 18, 1972, pp. 296-300.

An Efficient Method for Dynamic Response Optimization

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Introduction

OPTIMIZATION in mechanical and structural systems under dynamic loads has been of interest since the mid 1960's, not long after optimization under static loads was treated. A review of the literature on the subject is presented in Ref. 1 and several references cited therein. Methods of design sensitivity analysis for dynamic response (point-wise state variable) constraints have been studied recently.[1-5] In one approach, each such constraint is replaced by several constraints imposed at the local maximum point. This type of treatment is called the worst-case design formulation.[2-5] Numerical results show the formulation to be very effective as optimal solutions satisfying precise optimality conditions are obtained. The formulation alleviates difficulties with the equivalent functional constraint formulation of Refs. 5 and 6. Another treatment, called the hybrid formulation, is presented in Refs. 1 and 4. This treatment is to divide the domain of the constraint into several subdomains, each containing one local maximum point. An equivalent functional constraint is then formulated over certain subdomains around the local maximum point. The numerical results show that the hybrid formulation takes more iterations to converge and the optimum cost function is slightly higher as compared with the worst-case design formulation. However, the computer time is substantially reduced.

This Note continues to investigate the treatment of pointwise state variable constraints for the dynamic response optimization problems. A suggested alternate treatment is to impose constraints at the two grid points that bracket a local max-point. The main advantage of this formulation is that the points of local maxima for the constraint function need not be located very accurately. Much computational effort is expended in accurately locating the max-points, as in the worst case design formulation.[2] Therefore, the present formulation could result in an optimization algorithm that would be more efficient and effective than previous algorithms. The purpose of this Note is to study the performance of this procedure.

The general optimization problem for dynamic response considered here is defined in Eqs. (2.1)-(2.6) of Ref. 2. The ith dynamic response constraint is written as $\psi_i(z,b,t) \le 0$, $0 \le t \le T$ [Eq. (2.4) of Ref. 2], where b is a design parameter vector, z a state variable vector, and T the total time of interest. It is proposed to replace each such constraint by several constraints imposed at the time grid points bracketing a local max-point:

$$\psi_i(z,b,t_j) \le 0; \quad j=1, \quad 2m(i) \tag{1}$$

where t_j is a grid point neighboring the local maximum and $m(i)$ is the total number of max-points for ψ_i. Note that the number of constraints is twice the number of max-points. Therefore, the number of constraints to be imposed during the optimization process is twice that of the max-value or hybrid formulations. The proposed treatment contains all of the advantages of previous max-value[2] and hybrid treatments.[4] That is, it does not assume constant Lagrange function over the time domain, does not require precise location of local max-points, and does not require interpolation of certain (adjoint) variables during sensitivity analysis. It is therefore expected to be more efficient and effective than the previous treatments.

To compare the proposed treatment with equivalent functional max-value and hybrid treatments,[5,6] let us consider a constraint function ψ_i plotted in Fig. 1. The constraint is violated over three subdomains. The equivalent functional treatment would add areas I, II, and III to define a constraint. The max-value treatment would locate the max-point for each of the three subdomains and impose the constraint only at those points (i.e., points A, B, and C). A separate functional constraint for each of areas I, II, and III would be imposed in the hybrid treatment. In the proposed treatment, the constraint will be imposed at the time grid points neighboring each local max-point (i.e., points A_1, A_2, B_1, B_2, C_1, and C_2).

Design Sensitivity Analysis and Optimization Algorithm

To implement the proposed treatment in a numerical optimization method, we need to compute the design gradient of a constraint imposed at a particular time t_j. Two methods for calculating these gradients recently have been published[2]: direct differentiation and adjoint variable. Both methods require solution of additional differential equations. The direct differentiation method is such that sensitivity of a constraint function can be obtained at any time. With the adjoint variable method, a differential equation for each constraint in Eq. (1) must be defined and integrated. There are two different adjoint variable procedures. Depending on the numbers of design variables, constraints, and loading conditions, one method will be preferred over the other. The differential equation during sensitivity analysis can be linear with constant coefficients for a large class of nonlinear problems. The advantage of this fact is realized in computations.[1,2]

The computational optimization algorithm to implement the suggested treatment of dynamic constraints is very similar to that presented in Ref. 2. The only difference is in the number of constraints and the grid points at which they are imposed. All the numerical details given there also apply to the present treatment.

Sample Problems

To study the effectiveness of the proposed treatment of constraints, two design problems are optimized: a linear two-degree-of-freedom vibration isolator and a five-degree-of-freedom vehicle suspension system. The results are compared

Received March 30, 1984; revision received Sept. 28, 1984.

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with those obtained by earlier formulations. A direct differentiation method is also used to optimize problem 2 in example 1. Reference 5 may be consulted for detailed formulations of the problems. The optimization program and numerical methods used are the same as in Refs. 1, 2, and 4. All CPU times reported are for PRIME 750.

Example 1. Optimal Design of a Linear Two-Degree-of-Freedom Vibration Isolator

The design objective is to select the damping and spring constants that minimize the peak transient dynamic displacement of the main mass over a finite number of excitation frequencies for the forcing function, subject to constraints on transient and steady-state responses, and explicit bounds on design variables. Two design problems are solved: one excitation frequency and five excitation frequencies.

The numerical data for the problems are given in Refs. 2, 4, and 5. The solution for problem 1 with the direct differentiation method is as follows: design variables, 1.3277, 0.03062; cost function history, 3.189, 3.189, 1.991, 1.991, 2.300, 2.355; CPU time, 219.8 s. The active constraints are steady-state relative displacement between two masses and the transient response for the main mass at $t = 0.138$ and 0.518 s.

The solution for problem 1 with the adjoint variable method is as follows: design variables, 1.3277, 0.03059; cost function history, 3.189, 3.189, 1.844, 1.844, 1.844, 2.245, 2.351, 2.356; CPU time, 197.0 s; and active constraints are the same as above.

The solution for problem 2 with the adjoint variable method is as follows: design variables, 0.9212, 0.1543; cost function history, 9.844, 9.844, 9.844, 9.844, 5.422, 5.422, 5.422, 5.422, 4.049, 4.282, 4.293; CPU time, 986.5 s. The active constraints are the transient response of the main mass at $t = 0.852$, 0.250, 1.080 s for excitation frequency ratios[2,4,5] of 0.8, 1.0, and 1.1, respectively.

By comparing these results with those obtained previously,[1,2,4] it can be concluded that the optimal design for problem 1 is the same as obtained by the other treatments and that for design problem 2 is the same as obtained with the max-value treatment. The CPU time per iteration is less than that by the max-value treatment since precise location of the local max-points is not necessary for the present treatment. The CPU time per iteration is slightly higher than that by the hybrid treatment. This is expected because the number of constraints is twice that in the hybrid treatment.[4] However, the present formulation obtains the optimum solution in fewest iterations and the smallest CPU time among the three treatments. It is therefore concluded that the present treatment of point-wise state variable constraints is more efficient and effective than the previous treatments.

Example 2. Optimal Design for a Five-Degree-of-Freedom Vehicle Suspension System

The vehicle suspension system is to be designed to minimize the extreme acceleration of the driver's seat for a

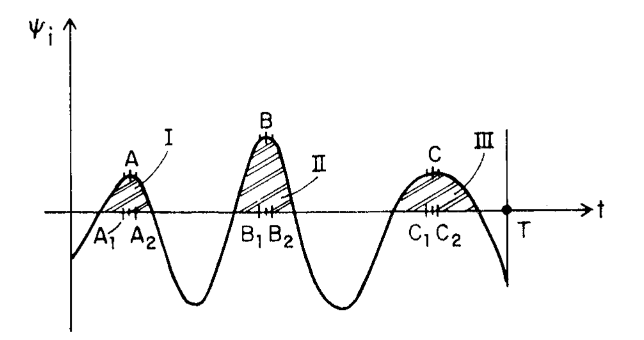

Fig. 1 Point-wise state variable constraint ψ_i, $0 \leq t \leq T$ violated over three time subdomains.

variety of vehicle speeds and road conditions. Spring constants and damping coefficients of the system are chosen as design parameters. The motion of the vehicle is constrained so that the relative displacements between the chassis and the driver's seat, the chassis and the front and rear axles, and the road surface and the front and rear axles are within given limits. Further, it is desirable to constrain seat acceleration due to an additional set of extreme road conditions. The design variables are also constrained. The problems considered here are exactly problems 1 and 3 of Ref. 5, which may be consulted for detailed derivations. The first design problem is for one road condition; the second one has two.

The numerical considerations, methods, and data for the problems are the same as in previous treatments. The total time interval is 5.0 s. The initial designs for two problems are [100, 300, 300, 10, 25, 25, 332.6] and [100, 300, 300, 10, 25, 25, 198.6], respectively. Lower and upper bounds for design variables are [50, 200, 200, 2, 5, 5, 1] and [500, 1000, 1000, 50, 80, 80, 500], respectively.

The solution for problem 1 with the adjoint variable method is as follows: design variables, 50.0, 252.2, 294.8, 20.95, 73.34, 80.0; cost function history, 332.6, 332.6, 332.6, 332.6, 332.6, 199.6, 199.6, 199.6, 199.6, 199.6, 246.1, 246.1, 246.1, 246.1, 260.4, 260.4; CPU time, 1409 s. The active constraints are the artificial acceleration constraint at 0.985 and 0.990 s, the relative displacement constraint between the chassis and the front axle at 1.050 and 1.055 s, the lower bound for design variable number one, and the upper bound for number six. The solution for problem 2 is as follows: design variables, 50.0, 200.0, 200.0, 25.56, 80.0, 80.0; cost function history, 198.6, 198.6, 198.6, 198.6, 198.6, 119.2, 119.2, 119.2, 119.2, 104.2, 104.2, 104.2, 80.06, 80.06, 80.06, 80.06, 84.63, 84.63, 84.63, 84.63, 99.02, 101.6, 100.3, 101.3; CPU time, 2928 s. The active constraints are the artificial acceleration constraint due to road profile No. 2 at 1.165 and 1.170 s, the lower bound for design variables 1, 2, and 3, and the upper bound for design variables 5 and 6.

Again, it can be seen that the new treatment takes less CPU time/iteration than the max-value treatment and more time than the hybrid treatment. The reason is the same as in the previous example. Note that the number of iterations and total CPU time are greatly reduced as compared with the max-value and hybrid treatments.[1,2,4] The results show that the new treatment is more efficient and effective than previous treatments. The optimum cost functions solved by the new treatment are slightly higher than those by previous treatments, because constraints are imposed only at the grid points. Therefore, the final design may depend on the size of time grid.

Discussion and Conclusions

An alternate treatment of point-wise state variable constraints is to impose the constraint at the two time grid points, each containing a local max-point. This treatment avoids the disadvantages of the functional, max-value, and hybrid treatments. It does not use the assumption of constant Lagrange function over a time domain as in the functional and hybrid treatments. In addition, it does not require precise location of local max-points and interpolation for certain variables in sensitivity analysis.[1,2,4]

The numerical results show that the method can satisfy a very strict optimization criterion. Due to the increase in the number of constraints, the CPU time per iteration with the new treatment is slightly higher than that with the hybrid treatment. However, the number of iterations and total CPU time are substantially reduced. It is therefore claimed that the present treatment is the most efficient and effective among all of the treatments. The final designs obtained with the new treatment are optimal for example 1 and near optimal for example 2. The local minimum point obtained for example 2 is slightly different because there are more constraints and they are imposed at different times. Finally, an excellent approach would be to employ the present treatment initially and then switch to the max-value approach near the optimal point.

Acknowledgment

This research is supported by the NSF Grant No. CEE 82-13851.

References

[1]Hsieh, C. C. and Arora, J. S., "Treatment of General Boundary Conditions and Point-Wise State Variable Constraints in Optimum Design for Static and Dynamic Response," Optimal Design Lab., College of Engineering, Univ. of Iowa, Iowa City, Rept. CAD-SS-84.1, Jan. 1984 (also available from National Technical Information Service, Springfield, Va., PB84-192749).

[2]Hsieh, C. C. and Arora, J. S., "Design Sensitivity Analysis and Optimization of Dynamic Response," *Computer Methods in Applied Mechanics and Engineering*, Vol. 43, 1984, pp. 195-219.

[3]Hsieh, C. C. and Arora, J. S., "Structural Design Sensitivity Analysis with General Boundary Conditions: Dynamic Problem," *International Journal for Numerical Methods in Engineering*, Vol. 21, No. 2, 1985, pp. 267-283.

[4]Hsieh, C. C. and Arora, J. S., "A Hybrid Formulation for Treatment of Point-Wise State Variable Constraints in Dynamic Response Optimization," *Computer Methods in Applied Mechanics and Engineering*, Vol. 48, 1985, pp. 171-189.

[5]Haug, E. J. and Arrora, J. S., *Applied Optimal Design*, John Wiley and Sons, Inc., New York, 1979.

[6]Feng, T. T., Arora, J. S., and Haug, E. J., "Optimal Structural Design under Dynamic Loads," *International Journal of Numerical Methods in Engineering*, Vol. 11, 1977, pp. 39-52.